APPROXIMATE SOLUTION OF MOMENTUM TRANSFER IN SYSTEM GENERALIZED NEWTONIAN FLUID–FLUIDIZED BED OF SPHERICAL PARTICLES USING MODIFIED RABINOWITSCH–MOONEY EQUATION

Vaclav DOLEJS, Petr MIKULASEK and Petr DOLECEK

Department of Chemical Engineering,
University of Pardubice, 532 10 Pardubice, Czech Republic

Received November 17, 1994
Accepted June 2, 1995

The modified Rabinowitsch–Mooney equation together with the corresponding relations for consistency variables has been adopted for approximate solution of momentum transfer between generalized Newtonian fluid with laminar flow and surface of fluidized bed of spherical particles inclusive of wall surface. The solution has been concretized for a fluid characterized by power-law and Ellis flow models in the creeping flow region. The range of values of ratios of particle diameter to column diameter and that of porosity values ε in which the suggested relation satisfactorily agrees with experimental results for pseudoplastic fluids have been delimitated experimentally.

Two exactly valid equations of Rabinowitsch–Mooney type, viz. for a flow through a tube[1,2] and that through a planar slot[3], are available for solving the problem of momentum transfer between a Generalized Newtonian Fluid (GNF) with laminar flow and a solid surface. Both these systems are characterized by a stress distribution which is independent of flow properties of GNF and, hence, is the same in the flow of a Newtonian Fluid (NF) and analogous flow of GNF. In systems of different geometry, such as, e.g., the flow of GNF through a fixed bed of particles[4,5], fall of particles in GNF (refs[6,7]), or flow through channels of noncircular cross section geometry[8], where the modified Rabinowitsch–Mooney equations are adopted, the agreement between stress distribution in NF flow and analogous GNF flow must be supposed. Then it is possible to approximately solve, using the results valid for NF, also the problems of GNF by the procedures described in refs[4–8].

The aim of the present work is to use the modified Rabinowitsch–Mooney equation also for the purposes of solution of momentum transfer in the system GNF–fluidized bed of spherical particles in the creeping flow region.

THEORETICAL

The paper[4] solves the problem of momentum transfer in the system GNF–fixed bed of particles by adopting the Rabinowitsch–Mooney equation,

$$D_w \equiv 2u_{ch}/l_{ch} = 4/\tau_w^3 \int_0^{\tau_w} \tau^2 \dot{D}(\tau)\, d\tau \quad , \tag{1}$$

where Eq. (*2*) was derived for the consistency variable τ_w with the use of theoretical idea of randomly arranged packing of particles which is defined[9] in such a way that the "planar" porosity (free area per unit of surface of planar cross section) is the same in any planar section of the bed and is equal to volume porosity, and with application of the integral scalar form of momentum balance[4] valid for the situation where the time change of momentum of fluid flowing through a reference area delimiting the investigated system is equal to zero:

$$\tau_w = \Delta p l_{ch}/L \quad . \tag{2}$$

The characteristic linear dimension of system, l_{ch}, for a fixed bed of spherical particles is given by the relation

$$l_{ch} = \varepsilon/[a_p\,(1-\varepsilon)\,(1+\psi) + a_w] = \varepsilon d/[6\,(1-\varepsilon)\,(1+\psi)\,M_w] \quad , \tag{3}$$

where the quantity ψ given by the ratio of shape and friction drag of bed of spherical particles has been called the resistance number, and M_w is a correction factor for the effect of walls given by the relation:

$$M_w = 1 + 4d/[6\,(1+\psi)\,(1-\varepsilon)\,D_h] \quad . \tag{4}$$

For a randomly arranged bed of particles, Eq. (*6*) follows for the characteristic velocity u_{ch} of the system from the equation of flow continuity in the form of Eq. (*5*)

$$S_b u_{ch} = S_c u \equiv \dot{V} \tag{5}$$

$$u_{ch} = u S_c/S_b \equiv u/\varepsilon \quad . \tag{6}$$

The Rabinowitsch–Mooney equation in its application has the meaning of approximately valid integral form of rheological state equation of GNF or it is exactly valid (tube, planar slot). This equation and the equation of flow continuity (*5*) only apply to an incompressible fluid (ρ = const).

Using the quantities ρ, u_{ch}, and τ_w, where τ_w represents the friction drag of unit area bypassed, we can express the Reynolds number defined generally as a ratio of characteristic magnitude of inertial forces of system to characteristic magnitude of friction forces by the following relation.

$$Re_{RM} = \rho u_{ch}^2 / \tau_w \qquad (7)$$

For a calculation of pressure drop of fixed bed, one must know the dependence of resistance criterion ψ on the Reynolds number Re_{RM} whose form must be determined experimentally[4]. The value ψ = 1/2 was derived[10] for the creeping flow region and for NF, which value is presumed[4] to apply to GNF, too.

Using Eqs (*3*) and (*6*), the consistency variables for a fixed bed of particles can be expressed by the relations:

$$D_w = 2u\,[6\,(1+\psi)\,(1-\varepsilon)]\,M_w / (d\varepsilon^2) \qquad (8)$$

$$\tau_w = \Delta p d\varepsilon / \left\{L\,[6\,(1+\psi)\,(1-\varepsilon)]\,M_w\right\} . \qquad (9)$$

In ref.[6] the Rabinowitsch–Mooney type equation in the form (*10*) was derived with the use of presumption of agreement of stress during flowing around a single spherical particle by NF and by GNF:

$$D_{w,p} \equiv 2u_{ch,p}/d = (\tau_{w,p}^{1/2}/2) \int_0^{\tau_{w,p}} \tau^{-3/2}\,\dot{D}(\tau)\,\mathrm{d}\tau \ . \qquad (10)$$

The difference in form between relation (*10*) and the Rabinowitsch–Mooney equation (*1*) is due to the different relations for distribution of shear stresses during flow through a tube and during flowing around a particle. For the tube, the shear stress increases with increasing value of dimensionless distance from the axis of cylindrical coordinate system, whereas for flowing around a single particle the shear stress decreases with increasing value of dimensionless distance from the centre of spherical coordinate system[6].

The equation (*10*) was also suggested[6] for the purposes of calculation of fall velocity of a particle in GNF provided the problem is symmetrical with respect to axis (in analogy to the Stokes approach[11]), i.e., if the particle does not perform the so-called secondary movement. If it does, then the momentum balance must be extended by the moment of momentum balance[12].

In the case of a fall of a particle, relation (*11*) follows for the consistency variable $\tau_{w,p}$ from the integral form of the balance:

$$\tau_{w,p} = (\rho_s - \rho)\, gd/[6\,(1 + \psi)]\ . \qquad (11)$$

On the basis of the presumptions introduced, the resistance number ψ has the same value in the creeping flow region as in the solution by Stokes[11] (ψ = 1/2). Here the characteristic velocity of system, $u_{ch,p}$, is the fall velocity of a single particle, u_g.

Next it is necessary to transform the consistency variables D_w and τ_w for fixed bed given by Eqs (*8*) and (*9*) and the correction factor for the effect of walls, M_w, given by relation (*4*) to obtain such forms which would also be satisfactory for a fluidized bed and which would be valid also for a single particle ($u = u_g$) at the conditions of $\varepsilon = 1$ and $d/D_h = 0$.

From the form of consistency variable D_w given by relation (*8*) it is obvious that its value is equal to zero for the above-given conditions. Therefore it is necessary to replace the form of dependence (which is present here) upon the porosity multiplied by the expression $6(1 + \psi)$ by another dependence, e.g., the simplest dependence of the type ε^a. This transformation must be carried out for a concrete value of porosity ε, which was done with the use of experimental results obtained with a random bed of spherical particles[10].

In ref.[10] it was stated that the system of Eqs (*1*), (*4*), (*8*), and (*9*) is satisfactory for the creeping flow of NF ($\psi = 1/2$, $\dot{D}(\tau) = \tau/\mu$, where μ is dynamic viscosity) with sufficient precision if the porosity of random fixed bed sufficiently precisely agrees with the value given by relation

$$\varepsilon_{ch} = 0.347 + 0.386\, d/D \qquad (12)$$

for $d/D \leq 0.25$.

Since the presumption of random arrangement of particles is fulfilled exactly for the condition $d/D = 0$ only, the porosity value of 0.347 represents the value corresponding to this arrangement. This value agrees very well with the value of 0.348 suggested by Denton[13] for a random fixed bed of spherical particles.

With the use of the porosity value of 0.347 corresponding to random arrangement of particles, the above-considered transformation equation will assume the form $0.347^a = 0.347^2/6(1 + \psi)\,(1 - 0.347)$ whose solution will give $a = 3.29 + 2.18 \log (1 + \psi)$.

In similar way we can transform also Eq. (*4*) for the correction factor for effect of walls, M_w, which looses its meaning for unit porosity. Here we will transform the whole expression reflecting the effect of walls, and solution of the equation in the form of $0.347^b = 4/[6(1 + \psi)\,(1 - 0.347)]$ will lead to $b = 2.18 \log(1 + \psi) - 0.020$.

Then the transformed kinematic consistency variable for fluidized bed can be expressed by relation

$$D_{w,f} = 2uM_{w,f}/[d\varepsilon^{3.29 + 2.18 \log(1+\psi)}] \ , \qquad (13)$$

where the transformed correction factor for effect of walls has the form:

$$M_{w,f} = 1 + d\varepsilon^{2.18 \log(1+\psi) - 0.020}/D_h \ . \qquad (14)$$

With the use of the presumption that there is no dissipation of mechanical energy due to mutual collisions between particles in a randomly arranged fluidized bed and the presumtion that the value of correction factor for effect of walls, M_w, is equal to one, we obtain Eq. (*15*) for the pressure drop from the force balance:

$$\Delta p = L\,(1 - \varepsilon)\,(\rho_s - \rho)\,g \ . \qquad (15)$$

On its introducing into Eq. (*9*) and replacing of correction factor M_w by factor $M_{w,f}$, Eq. (*16*) is obtained for use in solving a fluidized bed:

$$\tau_{w,f} = (\rho_s - \rho)\,gd\varepsilon/[6\,(1 + \psi)\,M_{w,f}] \ . \qquad (16)$$

Next we presume that Eqs (*13*), (*14*), and (*16*) obtained by transforming the relations valid for a fixed bed of particles, and simultaneously for a single particle ($\varepsilon = 1$, $M_{w,f} = 1$), will also be valid for the porosity region corresponding to fluidized bed up to certain maximum values of d/D_h ratio whose value must be determined experimentally as it was the case with random fixed bed.

The Rabinowitsch–Mooney equation (*1*) and the equation of Rabinowitsch–Mooney type (*10*) can be expressed by a single relation in the form

$$D_{w,f} = (3 + \Omega)/\tau_{w,f}^{2+\Omega} \int_0^{\tau_{w,f}} \tau^{1+\Omega}\,\dot{D}(\tau)\,d\tau \ , \qquad (17)$$

where $\Omega = 1$ for a flow through a tube and through an fixed bed[4], whereas $\Omega = -5/2$ for a flowed-around or falling particle[6].

Presuming that the dependence of dimensionless characteristic Ω upon porosity ε is linear, its value for a randomly arranged fixed bed[4] ($\varepsilon = 0.347$, $\Omega = 1$) and single particle either flowed-around or falling[6] ($\varepsilon = 1$, $\Omega = -5/2$) can be determined with the help of relation

$$\Omega = -5.36\varepsilon + 2.86 \;, \tag{18}$$

which is supposed to be also applicable to a porosity region corresponding to fluidized bed.

Using the presumption of agreement between stress distribution during flow of NF and GNF through a fluidized bed of particles, one can adopt Eq. (*17*), where the value of Ω is given by Eq. (*18*), for a fluidized bed, too, the value of ψ number in Eqs (*13*), (*14*), and (*16*) being again 1/2 in the creeping flow region.

As it follows from the given forms of equations of Rabinowitsch–Mooney type (*1*), (*10*), (*17*), these equations will be readily solved for such flow models in which the shear rate $\dot{D}$ is an explicit function of shear stress τ.

For the power-law flow model ($\dot{D}(\tau) = (\tau/K)^{1/n}$, where K and n are parameters of the model) the solution of Eq. (*17*) gives

$$D_{\mathrm{w,f}} = [n\,(3 + \Omega)/(1 + 2n + \Omega n)]\,(\tau_{\mathrm{w,f}}/K)^{1/n} \;, \tag{19}$$

whereas for the Ellis model ($\dot{D}(\tau) = \tau[1 + (\tau/\tau_{1/2})^{\alpha-1}]/\eta_0$, where η_0, $\tau/\tau_{1/2}$, and α are parameters of the model) it gives

$$D_{\mathrm{w,f}} = \tau_{\mathrm{w,f}}\,[1 + (3 + \Omega)\,(\tau_{\mathrm{w,f}}/\tau_{1/2})^{\alpha-1}/(2 + \alpha + \Omega)]/\eta_0 \;, \tag{20}$$

where the consistency variables are given by relations (*13*) and (*16*) in which the correction factor for effect of walls, $M_{\mathrm{w,f}}$, is given by expression (*14*).

The justifiability of the equation system (*13*), (*14*), (*16*), and (*18*)–(*20*) suggested for the purposes of solution of fluidized bed must be verified and delimited experimentally for NF as well as for the power-law and Ellis' fluids.

EXPERIMENTAL

Experimental results presented in ref.[14] were used for the verification of justifiability of the relations given in Theoretical above. The fall velocities of glass beads whose physical properties are given in Table I were measured in aqueous glycerol solutions and in aqueous solutions of hydroxyethylcellulose Natrosol 250 MR (Hercules Powder Comp., Netherland) and Cellosize QP 40 (Union Carbide Corp., U.S.A.), and in aqueous solution of methylcellulose Tylose MH 4000 (Hoechst, Germany). Columns of 8, 4, and 2 cm diameter were used for the measurements. In the experiments with fluidized bed, its height was determined as a function of volume flow rate of fluid. From the values found in this way, the experimental porosity value was obtained with the use of the relation $\varepsilon_{exp} = 1 - V_p/(LS_c)$, where the volume of particles, V_p, was determined from their weight and density.

The flow curves were measured with the help of a rotation rheometer with coaxial cylinders, Reotest 2. The zero shear rate viscosity η_0 of the model fluids was measured with a capillary viscosimeter of our own construction[15]. Tables II and III give the characteristics of fluids used and the parameters determined for the flow model used.

TABLE I
Physical properties of particles

Particle No.	d, mm	ρ_s, kg m^{-3}	Particle No.	d, mm	ρ_s, kg m^{-3}
1	1.465	2 506	3	2.024	2 515
2	1.923	2 527	4	2.949	2 514

TABLE II
Properties of adopted solutions and range of shear rate $\dot{D}$ used for determination of parameters of flow models (see Table III)

Solution No.	Composition	Density ρ, kg m^{-3}	$\dot{D}$, s^{-1}
1	92% Glycerol	1 240	–
2	0.6% Natrosol 250 MR	1 001	49–218
3	0.6% Natrosol 250 MR	1 001	49–218
4	0.8% Natrosol 250 MR	1 002	49–218
5	1.0% Natrosol 250 MR	1 002	9– 73
6	1.0% Natrosol 250 MR	1 002	9– 73
7	0.7% Tylose MH 4000	1 000	49–218
8	0.8% Tylose MH 4000	1 001	27–122
9	0.9% Tylose MH 4000	1 001	27–122
10	1.2% Tylose MH 4000	1 002	9– 73
11	1.6% Cellosize QP-40	1 003	49–218

RESULTS AND DISCUSSION

For evaluation of agreement between the results of our own experiments or those given in ref.[16] with the approach suggested, the mean relative deviation was used:

$$\delta = 1/N \sum_{i=1}^{N} |\delta_i| \ , \tag{21}$$

where the per cent relative deviation between the experimental value of porosity, ε_{exp}, and the calculated value ε_{calc} is given by the relation:

$$\delta_i = (\varepsilon_{exp}/\varepsilon_{cal} - 1) \ . \ 100\% \ . \tag{22}$$

Newtonian Fluid

Substitution for the consistency variables $D_{w,f}$ and $\tau_{w,f}$ using relations (*13*) and (*16*), where the correction factor for effect of walls, $M_{w,f}$, is given by relation (*14*), in Eq. (*19*),

TABLE III
Parameters of power-law flow model and Ellis flow model

Solution No.	Power-law model		Ellis model		
	n	K, Pa s^n	η_0, Pa s	$\tau_{1/2}$, Pa	α
1	1	0.273	0.273	–	1
2	0.76	0.141	0.067	9.33	2.29
3	0.86	0.112	0.081	6.62	2.00
4	0.78	0.235	0.162	10.1	1.84
5	0.70	1.01	1.04	5.02	2.00
6	0.70	1.06	1.11	4.95	1.92
7	0.82	0.176	0.94	28.5	2.16
8	0.88	0.254	0.200	26.6	1.98
9	0.84	0.283	0.204	24.8	1.85
10	0.88	0.770	0.75	27.4	1.82
11	0.93	0.130	0.140	96.0	1.92

where n = 1 and $K \equiv \mu$, after modification provides Eq. (*23*) for the creeping flow region (ψ = 1/2) and for column of circular cross-section ($D_h = D$):

$$(\rho_s - \rho)\, g d^2 \varepsilon^{4.67} / [\mu u\, (1 + d\varepsilon^{0.364}/D)^2] = 18 \quad . \tag{23}$$

This relation is, for the condition of d/D = 0, practically identical with that suggested by Lewis et al.[17] in which the experimentally determined numerical value of porosity exponent is 4.65. The same value was also experimentally obtained by Richardson and Zaki[18] and its justifiability is proved by other authors (e.g., refs[19–21]).

Moreover, results of our own experiments[14] and those by Andersson[16] were used to verify the justifiability of Eq. (*23*) and to determine the maximum value of d/D ratio delimiting the satisfactory fulfilment of the presumption of random arrangement of particles in fluidized bed. The results by Andersson were used because he dealt with the effect of d/D ratio within the largest range (0.025–0.308) and because these results are presented graphically in such scale that they can be read (and used) with sufficient accuracy.

The results of our own experiments (in the form of relative deviation) are given in Fig. 1 for the value of ratio d/D = 0.073. From the figure its can be seen that the deviations assume both positive and negative values, and a satisfactory agreement of the approach suggested with experiment is also evidenced by the low value of mean deviation δ = 0.7% for N = 22. The results by Andersson[16] show a similar agreement within the range of values $d/D \in$ (0.025; 0.052) where the value of deviation δ varied from 0.8 to 1.3%.

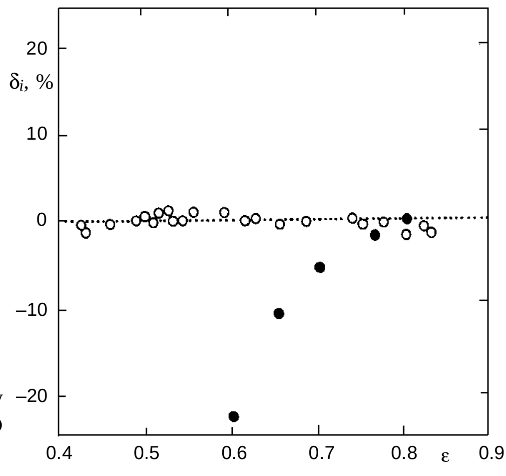

FIG. 1
Dependence of relative deviation δ_i on porosity ε: ❍ our own experiments for d/D = 0.073, ● experiments by Andersson[16] for d/D = 0.093

The experimental results by Andersson[16] for the next higher value of d/D measured by him are also given in Fig. 1. It can be seen that the absolute values of deviations are considerably higher as compared with the results of our own experiments for the value of ratio $d/D = 0.073$ and, beside that, they depend on the bed porosity, hence the approach suggested is no more satisfactory for the value of ratio $d/D = 0.093$.

The negative sign of deviations indicates that the given bed of particles has a lower resistance than a bed whose behaviour satisfactorily corresponds to the idea of randomly arranged bed. A similar phenomenon is encountered also with the fixed bed when the critical value of ratio $d/D = 0.25$ is exceeded and the non-arranged bed of particles is transformed into an arranged bed forming channels, which is connected with an abrupt decrease in drag of the bed[10].

The above-mentioned facts allow the conclusion that exceeding of the value of d/D ratio, found somewhere between the values 0.073 and 0.093, presumably results in formation of channels in a fluidized bed, too, the effect of these channels being the highest in the region of the lowest bed porosity values.

As there are no further experimental results available between the d/D ratio value measured by us (0.073) and that measured by Andersson[16] (0.093), let us preliminarily restrict the justifiability of the relation suggested to the values of ratio $d/D < 0.083$.

Moreover, the range of validity of the relation suggested should be limited by the minimum and the maximum porosity of fluidized bed whose values will obviously be best determined experimentally.

Results of our experiments carried out both in the creeping flow region and in region of manifestation of inertial forces of system[14] led to the value of $\varepsilon_{min} \approx 0.42$, whereas the results by Andersson[16] lead to the value of $\varepsilon_{max} \approx 0.93$.

Generalized Newtonian Fluid

Substitution of the consistency variables $D_{w,f}$ and $\tau_{w,f}$ using relations (*13*), (*14*), and (*16*) in relation (*19*) gives Eq. (*24*) for the creeping flow region ($\psi = 1/2$) and for a column of circular cross section ($D_h = D$):

$$2u\,(1 + d\varepsilon^{0.364}/D)/(d\varepsilon^{3.67}) =$$

$$= n\,(3 + \Omega)\left\{(\rho_s - \rho)\,gd\varepsilon/[9K\,(1 + d\varepsilon^{0.364}/D)]\right\}^{1/n}/(1 + 2n + \Omega n)\ . \qquad (24)$$

Analogous substitution in Eq. (*20*) and modification gives Eq. (*25*):

$$18u\eta_0\,(1 + d\varepsilon^{0.364}/D)^2/[(\rho_s - \rho)\,gd^{\,2}\varepsilon^{4.67}] =$$

$$= 1 + (3 + \Omega) \left\{(\rho_s - \rho)\, g d\varepsilon / [9\tau_{1/2}\, (1 + d\varepsilon^{0.364}/D)]^{\alpha-1}\right\} / (2 + \alpha + \Omega) \ . \qquad (25)$$

These relations along with Eq. (*18*) were used for the calculation of porosity ε_{calc}, using our experiments[14] with beds of particles in the range of values d/D < 0.083 for verification of relations (*24*) and (*25*). The characteristics of the systems used, namely the types of fluid and particles and the value of d/D ratio, are given in Table IV.

TABLE IV
Experimental results

Solution No.	Particle No.	d/D	N	δ_P, %	δ_E, %
2	1	0.073	8	2.8	3.3
3	1	0.037	15	2.9	2.9
		0.073	9	3.2	3.2
4	1	0.018	9	2.5	0.9
		0.037	8	3.0	3.1
		0.073	7	1.2	3.1
5	1	0.018	5	1.3	3.2
		0.037	3	2.1	5.3
		0.073	4	2.1	2.3
6	3	0.025	3	2.6	1.2
		0.050	3	0.8	2.2
7	1	0.073	16	4.8	1.2
8	4	0.037	14	2.6	1.9
		0.073	12	1.6	2.0
9	1	0.037	12	2.2	4.0
		0.073	10	1.3	3.2
9	4	0.074	13	1.6	2.1
10	1	0.018	9	1.5	0.9
		0.037	8	1.6	0.7
		0.073	7	3.0	2.2
11	2	0.049	19	1.6	1.6
			N = 194	δ_P = 2.3	δ_E = 2.3

Experimental results for one of the solutions with the lowest value of flow index $n = 0.70$ and three values of d/D ratio are given in Fig. 2 in the form of relative deviation δ_i. It can be seen that experimental results agree satisfactorily with the approach suggested up to the bed porosity value $\varepsilon \approx 0.6$. However, above this value, the magnitude of deviation increases with increasing porosity value and it depends on the value of d/D ratio. With regard to the negative sign of the deviation it is possible (as it was the case in the above-discussed effect of value of d/D ratio in the system NF–fluidized bed) to infer a bed again forming channels, which could readily be observed in this case[14].

A similar course of dependence of relative deviation δ_i upon porosity was found in other systems, too, the maximum porosity value ε_{max} (whose crossing is connected with formation of channels in the fluidized bed) being increased with increasing value of the flow index n.

The dependence of this maximum value of porosity, ε_{max}, on the value of flow index n can approximately be expressed by the relation

$$\varepsilon_{max} = 0.93 - 0.96\,(1 - n)^{0.75} \qquad (26)$$

for $0.7 < n < 0.93$, where the numerical value 0.93 represents the maximum value of porosity of fluidized bed for a Newtonian fluid ($n = 1$).

The course of this dependence together with experimental ε_{max} values is given in Fig. 3 for all the fluids given in Table III.

Whereas a power-law fluid is transformed into Newtonian fluid for $n = 1$, where $K = \mu$, Ellis' fluid is transformed into Newtonian fluid for $\alpha = 1$ and $\tau_{1/2} \to \infty$, where $\eta_0 = \mu$.

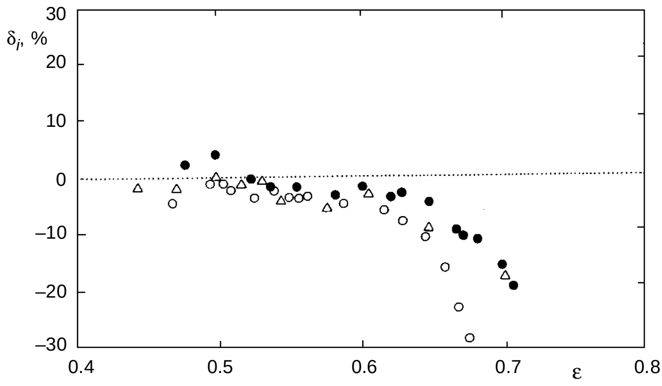

FIG. 2
Dependence of relative deviation δ_i on porosity ε for solution of Natrosol 250 MR with flow index $n = 0.70$: ❍ $d/D = 0.073$, ● $d/D = 0.037$, Δ$d/D = 0.018$

On the basis of this comparison it is expected that the maximum porosity ε_{max} of Ellis' fluid will depend on the parameters α and $\tau_{1/2}$ of the Ellis model. This dependence can approximately be expressed by the relation,

$$\varepsilon_{max} = 0.93 - 0.70\tau_B^{-0.19\alpha} \quad (27)$$

for $1.6 \cdot 10^{-4} < \tau_B^{-\alpha} < 4.6 \cdot 10^{-2}$. Here $\tau_B = \tau_{1/2}/\tau_1$, where $\tau_1 = 1$ Pa.

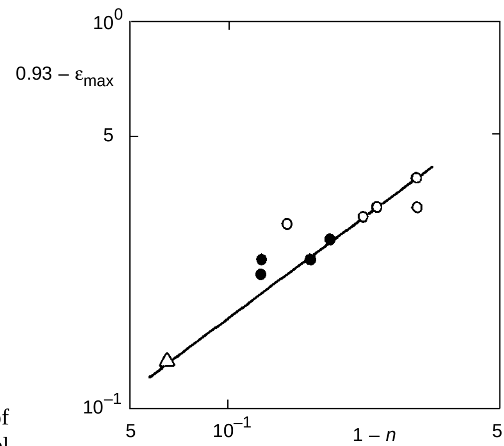

FIG. 3
Dependence of maximum value of porosity of fluidized bed ε_{max} on flow index n: ❍ Natrosol 250 MR, ● Tylose MH 4000, Δ Cellosize QP-350, —— Eq. (*26*)

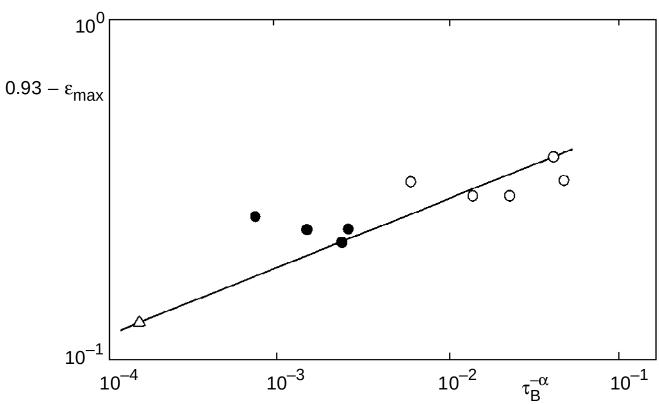

FIG. 4
Dependence of maximum value of porosity of fluidized bed, ε_{max}, on value of $\tau_B^{-\alpha}$ quantity: ❍ Natrosol 250 MR, ● Tylose MH 4000, Δ Cellosize QP-350, —— Eq. (*27*)

The course of this dependence together with experimental values of ε_{max} is given in Fig. 4.

Beside the maximum porosity value also the minimum porosity value must be given to delimit satisfactory validity of relations (*24*) and (*25*). From the experimental results[14] it was possible to determine the mean value of the minimum porosity $\varepsilon_{min} \approx 0.46$, which is higher than the value of 0.42 found for NF.

Table IV gives (in the form of mean relative deviations for all the systems fluid–particle–column) the experimental results for the porosity values ε lower than those obtained from relations (*26*) and (*27*).

From the table it is obvious that the values of mean relative deviations for the whole data set are identical for the power-law and Ellis models: For the solutions of hydroxyethylcellulose Natrosol and methylcellulose Tylose, the mean relative deviations are identical (2.4%) in the power-law model, whereas in the Ellis model the deviations with Natrosol solutions (2.8%) are higher than those with Tylose solutions (2.0%).

Hence the experiments with fluidized bed of spherical particles lead to the same conclusion as those with fixed bed of particles[5] where more complex flow models did not offer any advantage over the simple power-law model either, and where the magnitude of deviation from the approach suggested also depended on the type of model fluid used.

The approach suggested has an advantage over the procedures used so far (e.g., refs[22–28]) in that it is general. It is formulated independently of any concrete flow model of GNF and it also involves the effect of walls, no empirical corrections being needed in the relations suggested.

We also expect that the relations suggested will be applicable to the regions of manifestation of effect of Reynolds number provided the concrete form of the dependence of $\psi = \psi\,(Re_{RM})$ is known for the fluidized bed. Its course will have to be determined (as it is the case in the approach to flow through a fixed bed[4]) from experimental results obtained with NF and fluidized bed of spherical particles.

CONCLUSION

A qualitatively and quantitatively new approach has been suggested to the momentum transfer in systems of GNF and fluidized bed of spherical particles. It is based on application of a Rabinowitsch–Mooney type equation together with the corresponding relations for the consistency variables and for the fluidized bed. Its satisfactory validity has been verified and delimited experimentally in the creeping flow region both for NF and for pseudoplastic GNF characterized by the power-law flow model and Ellis' flow model.

SYMBOLS

a	exponent for transformation of consistency variable D_w
a_p	$= 6/d$ specific surface of spherical particle, m^{-1}
a_w	$= 4/D_h$ specific surface of walls, m^{-1}
b	exponent for transformation of factor of effect of walls M_w
D	column diameter, m
$\dot{D}$	shear rate, s^{-1}
D_h	hydraulic diameter of column, m
D_w	consistency variable for flow through a tube and fixed bed of particles, s^{-1}
$D_{w,p}$	consistency variable for fall of spherical particle, s^{-1}
$D_{w,f}$	consistency variable for fluidized bed, s^{-1}
d	diameter of spherical particle, m
g	acceleration of gravity, m s^{-2}
K	parameter of power-law model, Pa s^n
L	bed height, m
l_{ch}	characteristic linear dimension of system for flow through a tube and fixed bed, m
M_w	correction factor for effect of walls in the case of fixed bed
$M_{w,f}$	correction factor for effect of walls in the case of fluidized bed
N	number of experiments
n	parameter of power-law model
Δp	pressure drop, Pa
Re_{RM}	Reynolds number of Rabinowitsch–Mooney type, Eq. (*7*)
S_c	cross section of column, m^2
S_b	cross section of the bed, m^2
u	mean superficial velocity, m s^{-1}
u_g	fall velocity of a single particle, m s^{-1}
u_{ch}	characteristic velocity of system for flow through tube and fixed bed, m s^{-1}
$u_{ch,p}$	characteristic velocity of system for fall of a single particle, m s^{-1}
$\dot{V}$	volumetric flow rate, m^3 s^{-1}
V_p	volume of particles in bed, m^3
α	parameter of Ellis model
δ	mean relative deviation
δ_i	relative deviation of an individual measurement
ε	$= S_b/S_c$ porosity
ε_p	the minimum bed porosity
μ	dynamic viscosity, Pa s
ρ	density of fluid, kg m^{-3}
ρ_s	density of particle, kg m^{-3}
τ	shear stress, Pa
τ_B	dimensionless value of parameter of Ellis model $\tau_{1/2}$
τ_w	consistency variable for flow through a tube and fixed bed, Pa
$\tau_{w,p}$	consistency variable for fall of spherical particle, Pa
$\tau_{w,f}$	consistency variable for flow through fluidized bed, Pa
$\tau_{1/2}$	parameter of Ellis model, Pa
η_0	parameter of Ellis model, Pa s
ψ	resistance number
Ω	dimensionless parameter, Eq. (*18*)

Indexes

calc	calculated
ch	characteristic
exp	experimental
max	maximum
E	Ellis model
P	power-law model

REFERENCES

1. Rabinowitsch B.: Z. Phys. Chem., A *145*, 1 (1929), taken from ref.[29].
2. Mooney M.: J. Rheol. *2*, 210 (1931), taken from ref.[29].
3. Smolskii B. M., Shulman Z. P., Gorislavets V. M.: *Reodinamika i teploobmen nelineino-vyazkoplastichnykh materialov*. Izd. Nauka i Techn., Minsk 1969.
4. Dolejs V., Lecjaks Z.: Chem. Prum. *34*, 449 (1978); Int. Chem. Eng. *20*, 466 (1980).
5. Machac I., Dolejs V.: Chem. Eng. Sci. *36*, 1679 (1981).
6. Dolejs V.: Sb. Ved. Pr. Vys. Sk. Chem.-Technol. Pardubice *48*, 175 (1985).
7. Dolejs V.: Collect. Czech. Chem. Commun. *58*, 1525 (1993).
8. Dolejs V., Machac I., Dolecek P.: Collect. Czech. Chem. Commun. *59*, 603 (1994).
9. Debbas S., Rumpf H.: Chem. Eng. Sci. *21*, 583 (1966).
10. Dolejs V.: Chem. Prum. *27*, 275 (1977); Int. Chem. Eng. *18*, 718 (1978).
11. Bird R. B., Stewart W. E., Lightfoot E. N.: *Transport Phenomena*, p. 56. Wiley, New York 1960.
12. Lecjaks Z., Pantoflicek J., Hac J., Volsik V.: Chem. Prum. *19*, 206 (1969).
13. Denton W. H.: *United Kingdom Atomic Energy Research Establishment Report E/R* 1095 (1957), taken from ref.[30].
14. Mikulasek P.: *Ph.D. Thesis*. University of Chemical Technology, Pardubice 1991.
15. Mikulasek P., Cakl J.: Sb. Ved. Pr. Vys. Sk. Chem.-Technol. Pardubice *56*, 235 (1992/93).
16. Andersson K. E. B.: Chem. Eng. Sci. *15*, 276 (1961).
17. Lewis W. E., Gilliland E. R., Bauer W. C.: Ind. Eng. Chem. *41*, 1104 (1949).
18. Richardson J. F., Zaki W. N.: Trans. Instn. Chem. Eng. *32*, 35 (1954).
19. Bena J., Ilavsky J., Kossaczsky E., Neuzil L.: Collect. Czech. Chem. Commun. *28*, 293 (1963).
20. Godard K., Richardson J. F.: Chem. Eng. Sci. *24*, 363 (1969).
21. Davidson J. F., Harrison D.: *Fluidisation*. Academic Press, London and New York 1971.
22. Yu Y. H., Wen C. Y., Bailie R. C.: Can. J. Chem. Eng. *46*, 149 (1968).
23. Mishra P., Singh D., Mishra J. M.: Chem. Eng. Sci. *30*, 397 (1975).
24. Brea F. M., Edwards M. F., Wilkinson W. L.: Chem. Eng. Sci. *31*, 329 (1976).
25. Kumar S., Upadhyay S. N.: Ind. Eng. Chem. Fundam. *20*, 186 (1984).
26. Mohan V., Raghuraman.: AIChE J. *22*, 259 (1976).
27. Kawase Z., Ulbrecht J.: Chem. Eng. Sci. *36*, 1193 (1981).
28. Machac I., Balcar J., Lecjaks Z.: Chem. Eng. Sci. *41*, 591 (1986).
29. Ulbrecht J., Mitschka P.: *Chemicke inzenyrstvi nenewtonskych kapalin*. Academia, Praha 1965.
30. Gray W. H.: *The Packing of Solid Particles*, p. 29. Chapman and Hall, London 1968.